

Effective Kähler potential and extra space-time dimensions

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The effects of extra space-time dimensions on the Wilsonian effective Kähler potential and the perturbative one loop effective Kähler potential are determined within the framework of an Abelian gauge theory with $N = 2$ supersymmetric field content. The relation between the Kähler metric and the effective gauge couplings which leads to the absence of radiative corrections to the Kähler potential is expressed as a function of the radius of compactification of a fifth dimension. In general, the quantum corrections to the low energy Kähler potential are shown to grow with this radius reflecting the underlying higher dimensional nature of the theory. [S0556-2821(99)02714-9]

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I. INTRODUCTION

As a consequence of the large degree of symmetry present in supersymmetric theories, the form of their low energy Wilsonian effective actions is highly constrained. For instance, when $N=1$ SUSY is combined with the required holomorphic dependence on the fields and parameters of a model, the superpotential can be completely (non-perturbatively) determined even in the presence of strongly interacting gauge fields [1]. Furthermore, for extended $N = 2$ supersymmetric theories, the entire low energy action can be constructed in terms of a single holomorphic prepotential. As in the $N=1$ superpotential case, this prepotential is exactly determined using the symmetries, holomorphicity and duality properties of the model in question [2]. The simplest $N=2$ gauge theory consists of an $N=1$ $U(1)$ gauge superfield V and its neutral chiral φ and antichiral $\bar{\varphi}$ superfield $N=2$ partners. In terms of these $N=1$ superfields, the low energy effective action has the form

$$\begin{aligned} \Gamma_{N=2}[\varphi, \bar{\varphi}, V] = & \frac{1}{8\pi i} \left(\int dV [\mathcal{F}_{\varphi}(\varphi) \bar{\varphi} - \bar{\mathcal{F}}_{\bar{\varphi}}(\bar{\varphi}) \varphi] \right. \\ & + \frac{1}{2} \int dS \mathcal{F}_{\varphi\varphi}(\varphi) W^{\alpha} W_{\alpha} \\ & \left. - \frac{1}{2} \int d\bar{S} \bar{\mathcal{F}}_{\bar{\varphi}\bar{\varphi}}(\bar{\varphi}) \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} \right). \end{aligned} \quad (1.1)$$

Here subscripts denote differentiation with respect to that variable so that, for example, $\mathcal{F}_{\varphi} = \partial \mathcal{F} / \partial \varphi$. The chiral field strength is defined by $W_{\alpha} = -\frac{1}{4} \bar{D} \bar{D} e^{-2V} D_{\alpha} e^{2V} = -\frac{1}{2} \bar{D} \bar{D} D_{\alpha} V$ with a similar definition for the anti-chiral field strength: $\bar{W}_{\dot{\alpha}} = -\frac{1}{2} D D \bar{D}_{\dot{\alpha}} V$. The holomorphic prepotential $\mathcal{F}(\varphi)$ then determines the Kähler potential $K(\varphi, \bar{\varphi}) = \text{Im}(\mathcal{F}_{\varphi}(\varphi) \bar{\varphi})$ and hence the Kähler metric $g(\varphi, \bar{\varphi}) = K_{\varphi \bar{\varphi}} = \text{Im} \mathcal{F}_{\varphi\varphi}$ as well as the effective gauge couplings: $\mathcal{F}_{\varphi\varphi}$, $\bar{\mathcal{F}}_{\bar{\varphi}\bar{\varphi}}$. The self-radiative corrections to this general $N = 2$ supersymmetric Abelian gauge theory action were found to vanish [1,3,4].

Extra compact space dimensions are a common attribute of many currently proposed fundamental theories. Concomi-

tant with these is the tower of Kaluza-Klein (K-K) field excitations whose masses are integer multiples of the inverse compactification radius R . In models where R is the order of the inverse Planck scale or smaller, the K-K modes unconditionally decouple from the low energy physics. On the other hand, for radii much larger than the inverse Planck scale, low energy physics can be considerably affected by the propagation of fields into the large radii extra dimensions. At the very least, theories formulated in higher dimensions tend to have a more divergent short distance behavior which leads to the breakdown of renormalizability. Thus an ultraviolet cutoff Λ is necessary for a consistent interpretation of such field theories. For large compactification radii, $\Lambda R > 1$, the K-K modes with mass below the cutoff can effect the low energy physics [5–9]. The resulting theoretical and phenomenological consequences include, for example, the possibility of introducing [10–13] a consistent and calculable mechanism of supersymmetry breaking and/or communicating the supersymmetry breaking between different four dimensional boundaries [14,15] and could also be instrumental in the description of the strong coupling limit of string theory [16]. In addition, their influence on the running of the gauge coupling and its effect on the scale of unification has been investigated [17]. It has even been suggested that they could lead to possible experimentally accessible modifications to gravitational interactions in the millimeter range [18].

In four dimensions, the general low energy action containing terms through two space-time derivatives for a $N = 1$ supersymmetric theory with the $N=2$ field content of an Abelian gauge superfield V and a neutral (anti-)chiral field ($\bar{\varphi}$) φ , has the form

$$\begin{aligned} \Gamma[\varphi, \bar{\varphi}, V] = & \int dV K(\varphi, \bar{\varphi}) + \frac{1}{2} \int dS f(\varphi) W^{\alpha} W_{\alpha} \\ & + \frac{1}{2} \int d\bar{S} \bar{f}(\bar{\varphi}) \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} + \int dS P(\varphi) \\ & + \int d\bar{S} \bar{P}(\bar{\varphi}) + \kappa \int dV V. \end{aligned} \quad (1.2)$$

Here the Kähler potential, $K(\varphi, \bar{\varphi})$, the effective gauge couplings, $f(\varphi)$ and $\bar{f}(\bar{\varphi})$, and the superpotentials, $P(\varphi)$ and

$\bar{P}(\bar{\varphi})$, are arbitrary, unrelated functions of their respective arguments. The linear in V Fayet-Iliopolous term has an arbitrary coupling constant, κ , which as a consequence of gauge invariance is necessarily field independent. Independent radiative corrections vanish for the superpotential in accord with the non-renormalization theorem in this topologically trivial model. Likewise, the Fayet-Iliopolous term has no independent additive radiative corrections. Hence each of these terms remain zero if they initially vanish. Finally, while the effective gauge couplings $f(\varphi)$ and $\bar{f}(\bar{\varphi})$ also remain uncorrected, the Kähler potential does receive radiative corrections. Setting the Fayet-Iliopolous terms to zero, these quantum effects are given by the Wilson renormalization group equations (WRGE) [19–21] for $K, f(\bar{f})$ and $P(\bar{P})$ as [3]

$$\begin{aligned}\frac{\partial K}{\partial t} &= 2K - (1 - \gamma)\varphi K_{\varphi} - (1 - \gamma)\bar{\varphi} K_{\bar{\varphi}} + \frac{1}{8\pi^2} \ln \left[\frac{g}{f + \bar{f}} \right] \\ \frac{\partial f}{\partial t} &= 2\gamma_V f - (1 - \gamma)\varphi f_{\varphi}; \quad \frac{\partial \bar{f}}{\partial t} = 2\gamma_V \bar{f} - (1 - \gamma)\bar{\varphi} \bar{f}_{\bar{\varphi}} \\ \frac{\partial P}{\partial t} &= 3P - (1 - \gamma)\varphi P_{\varphi}; \quad \frac{\partial \bar{P}}{\partial t} = 3\bar{P} - (1 - \gamma)\bar{\varphi} \bar{P}_{\bar{\varphi}},\end{aligned}\quad (1.3)$$

where all quantities have been made dimensionless by scaling with the loop momentum cutoff Λ , so that, for example $K \rightarrow K/\Lambda^2$ and $\varphi \rightarrow \varphi/\Lambda$. The result was obtained using the R_{ξ} gauge fixing action term

$$\Gamma_{\xi}[V] = \xi \int dV D D V \bar{D} \bar{D} V \quad (1.4)$$

and included all graphical contributions through two space-time derivatives and arbitrary powers of the fields. In this simple Abelian model, the gauge fixing parameter ξ receives no radiative corrections nor contributes to the other functions.

Since the fields are rescaled according to their wave function renormalization in order to maintain a canonical kinetic energy term as the short distance degrees of freedom are integrated out of the theory, the chiral field anomalous dimension γ can be found by evaluating the renormalization group equation for the Kähler metric, $g = K_{\varphi\bar{\varphi}}$, at $\varphi = 0 = \bar{\varphi}$ where it is normalized as $g|_{\varphi=0=\bar{\varphi}} = 1$. Likewise, the photon anomalous dimension, γ_V , is secured using the renormalization group equation for f and \bar{f} evaluated at zero fields $\varphi = 0 = \bar{\varphi}$ along with the normalizations $f|_{\varphi=0} = \frac{1}{2} = \bar{f}|_{\bar{\varphi}=0}$. So doing, one finds that $\gamma_V = 0$ and

$$2\gamma = -\frac{1}{8\pi^2} [g_{\varphi\bar{\varphi}} - g_{\varphi} g_{\bar{\varphi}} + f_{\varphi} \bar{f}_{\bar{\varphi}}]|_{\varphi=0=\bar{\varphi}}. \quad (1.5)$$

If the initial choice of Kähler potential and effective gauge coupling satisfies the $N=2$ supersymmetric relation $g=f$

$+\bar{f}$, then the anomalous dimension of the chiral field vanishes and the Wilson renormalization group equation for K reduces to that of naive scaling. There are no radiative corrections to the Wilson effective action.

One can also compute the corrections using ordinary perturbation theory. The one loop correction to the dimensionless effective Kähler potential is found to be simply given by

$$\delta K_{1\text{-loop}} = -\frac{1}{16\pi^2} \ln \left[\frac{g}{f + \bar{f}} \right]. \quad (1.6)$$

Just as was the case in the WRGE analysis, the $N=2$ supersymmetric relation $g=f+\bar{f}$ leads to no one-loop radiative corrections [1,24–26].

The purpose of this paper is to determine the effects of an extra compact dimension on the supersymmetric Wilsonian effective action as well as its effects on the perturbative one-loop effective action. Starting with an $N=1$ SUSY theory with a compact fifth dimension and Fourier expanding the fields over the circular fifth dimension results in the four dimensional theory with the $N=2$ SUSY field content of the above described zero mode fields, V , φ , and $\bar{\varphi}$, as well as the infinite tower of massive K-K $N=2$ SUSY multiplets, $\{V_n, \phi_n, \bar{\phi}_n\}$, for n any non-zero (positive and negative) integer. The resultant four dimensional action can thus be written as an expansion in powers of the K-K tower fields as

$$\Gamma = \sum_{l=0}^{\infty} \Gamma_l. \quad (1.7)$$

Here Γ_0 contains zero mode fields only. Due to momentum conservation in the direction of the fifth dimension, $\Gamma_1 = 0$, while Γ_2 contains 2 K-K tower fields, and so on. We seek to construct the Wilson renormalization group equation for the zero mode field action, $\Gamma_0[V, \varphi, \bar{\varphi}]$ after inclusion of the radiative effects of K-K towers. In addition, the perturbative one loop radiative corrections to Γ_0 will be calculated.

In both cases, the quantum corrections arise from the self-radiative contributions from Γ_0 as well as from radiative corrections of Γ_2 to Γ_0 . In either case, the form of Γ_2 is required for the determination of the quantum corrections. In a complete Wilson renormalization group analysis, the evolution of Γ_2 is determined by itself and Γ_4 and in turn Γ_4 is determined by contributions from Γ_6 and so on. Similarly, higher order perturbative contributions to Γ_0 come from higher numbers of K-K tower fields in the action. Since such an infinitely iterative procedure is beyond our calculational means, we must make additional assumptions regarding the form of the Γ_0 and Γ_2 actions before we can proceed. First of all, we neglect all higher derivative terms than those appearing in the kinetic energy. This constitutes the next to leading order in a derivative expansion of the action and allows for the determination of the anomalous dimensions. It thus constitutes an improvement beyond the local potential approximation [22] which is possible [23] because the kinetic terms arise from a Kähler potential. Thus for Γ_0 , we use Eq. (1.2) without the Fayet-Iliopoulos term ($\kappa=0$). A

similar form is assumed for the Γ_2 action except that explicit compactified radius dependent mass terms are included for the $N=2$ K-K tower fields. Thus we take

$$\begin{aligned} \Gamma_2 = & \sum_{\substack{n=-\infty \\ n \neq 0}}^{+\infty} \left\{ \int dV [g(\varphi, \bar{\varphi}) \bar{\phi}_n \phi_n + M_n^2 V_n V_{-n}] \right. \\ & + \int dS \left[\frac{1}{4} f(\varphi) W_n^\alpha W_{-n\alpha} + m_n(\varphi) \phi_n \phi_{-n} \right] \\ & \left. + \int d\bar{S} \left[\frac{1}{4} \bar{f}(\bar{\varphi}) \bar{W}_{n\dot{\alpha}} \bar{W}_{-n}^{\dot{\alpha}} + \bar{m}_n(\bar{\varphi}) \bar{\phi}_n \bar{\phi}_{-n} \right] \right\}. \end{aligned} \quad (1.8)$$

The K-K vector field mass term is given by $M_n^2 = n^2/R^2$ while the K-K (anti-) chiral field mass term is $[\bar{m}_n(\bar{\varphi}) = n/R - i\bar{P}_{\bar{\varphi}\bar{\varphi}}]$ $m_n(\varphi) = n/R + iP_{\varphi\varphi}$, where $n = \pm 1, \pm 2, \dots, \pm\infty$. The form of the action Γ_2 corresponds to a radical truncation of all generally possible terms. It has the property that all modes have the same, isotropic four dimensional action coefficients. Even in the large R limit, it is only the terms with momentum in the fifth dimension that are treated anisotropically. Of course, this is by no means a justification of the truncation which can only be achieved by a detailed analysis of the compactification of the more fundamental SUSY theory involved. This is quite a lengthy task and is beyond the scope of this present investigation.

II. THE WILSON EFFECTIVE KÄHLER POTENTIAL

The Wilson renormalization group equation governing the evolution of the zero mode Kähler potential is determined by integrating out the degrees of freedom with momentum in an infinitesimal four dimensional shell about $\Lambda(t) = e^{-t}\Lambda$. Although we are summing over all K-K modes in this shell, those with mass above the scale $\Lambda(t)$ decouple. As the sum over these modes takes place, the action is rescaled according to the wavefunction renormalization of the fields. Fixing the zero field metric at the scale $\Lambda(t)$ to its canonically normalized value of one, $g|_{\varphi=0=\bar{\varphi}} = 1$, and further rescaling all dimensionful quantities by the appropriate factors of Λ needed to render them dimensionless, the Wilson renormalization group equation for the zero mode Kähler potential is obtained as

$$\begin{aligned} \frac{\partial K}{\partial t} = & 2K - (1-\gamma)\varphi K_\varphi - (1-\gamma)\bar{\varphi} K_{\bar{\varphi}} \\ & + \frac{1}{16\pi^2} \sum_{n=-\infty}^{+\infty} \ln \left[\left(\frac{g^2 + r + \frac{n^2}{R^2}}{(f+\bar{f}) + \frac{n^2}{R^2}} \right)^2 \right] \\ & \times \left(\frac{\left[1 + \frac{n^2}{R^2} \right]^2}{1 + r(0) + \frac{n^2}{R^2}} \right). \end{aligned} \quad (2.1)$$

Here the dimensionless mass coefficient is $r = P_{\varphi\varphi} \bar{P}_{\bar{\varphi}\bar{\varphi}}$, with $r(0)$ the value of the dimensionless mass parameter at zero field, $r(0) = r|_{\varphi=0=\bar{\varphi}}$. The effective gauge couplings and superpotential remain uncorrected and their Wilson renormalization group equations are once again given in equation (1.3). The sum over the K-K modes may now be performed yielding

$$\begin{aligned} \frac{\partial K}{\partial t} = & 2K - (1-\gamma)\varphi K_\varphi - (1-\gamma)\bar{\varphi} K_{\bar{\varphi}} \\ & + \frac{1}{8\pi^2} \ln \left[\frac{\sinh[\pi R \sqrt{g^2 + r}]}{\sinh[\pi R \sqrt{1 + r(0)}]} \frac{\sinh^2[\pi R]}{\sinh^2[\pi R \sqrt{f+\bar{f}}]} \right], \end{aligned} \quad (2.2)$$

where g and f, \bar{f} have been normalized at zero field to be 1 and 1/2, 1/2, respectively.

The four dimensional limit, $R \rightarrow 0$, reproduces the Wilson renormalization group Eq. (1.3) obtained previously. For $R \gg 1$, the Wilson renormalization group equation for K reduces to

$$\begin{aligned} \frac{\partial K}{\partial t} = & 2K - (1-\gamma)\varphi K_\varphi - (1-\gamma)\bar{\varphi} K_{\bar{\varphi}} \\ & + \frac{2\pi R}{16\pi^2} [\sqrt{g^2 + r} - 2\sqrt{f+\bar{f}} - \sqrt{1 + r(0)} + 2]. \end{aligned} \quad (2.3)$$

This can be recast by further rescaling the fields and the Kähler potential according to their five dimensional engineering dimensions, $\varphi \rightarrow \varphi/\sqrt{2\pi R}$, $\bar{\varphi} \rightarrow \bar{\varphi}/\sqrt{2\pi R}$, $K \rightarrow (1/2\pi R)K$, as

$$\begin{aligned} \frac{\partial K}{\partial t} = & 3K - \left(\frac{3}{2} - \gamma \right) \varphi K_\varphi - \left(\frac{3}{2} - \gamma \right) \bar{\varphi} K_{\bar{\varphi}} \\ & + \frac{1}{16\pi^2} [\sqrt{g^2 + r} - 2\sqrt{f+\bar{f}} - \sqrt{1 + r(0)} + 2]. \end{aligned} \quad (2.4)$$

This result can also be obtained directly in five dimensions by integrating over the fifth dimension momentum variable rather than performing a Fourier series.

For $R \neq 0$, the radiative corrections to the Kähler potential do not, in general, vanish at the four dimensional, $N=2$ relation between the metric and effective gauge coupling. Rather one finds that, with $r=0$, the logarithm on the right hand side of Eq. (2.2) vanishes resulting in no radiative correction to the Kähler potential, K , provided

$$\frac{\sinh[\pi R] \sinh[\pi R g]}{\sinh^2[\pi R \sqrt{f+\bar{f}}]} = 1. \quad (2.5)$$

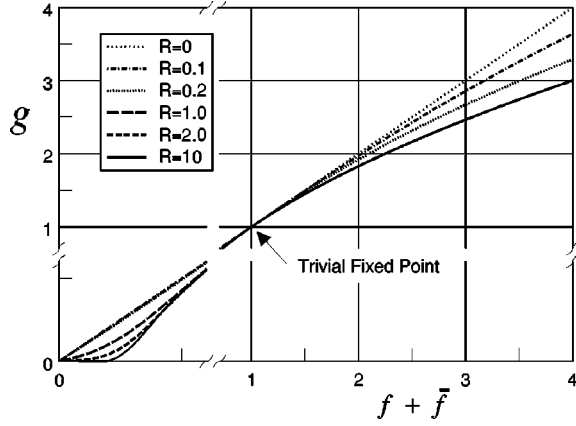


FIG. 1. The relation between g and $f + \bar{f}$ resulting in the absence of radiative corrections for various compactification radii.

When this condition is satisfied, the Wilson renormalization group equation yields the naive scaling equation for the Kähler potential.

The four dimensional $N=2$ relation between metric and effective gauge couplings that leads to the absence of radiative corrections is $g = (f + \bar{f})$, which is also the $R \rightarrow 0$ limit of Eq. (2.5). In Fig. 1, we plot the relation between g and $f + \bar{f}$ for different compactified radii, R , values which leads to no radiative corrections. We see that, due to the exponential dependence on R , the no radiative correction condition quickly deviates from the four dimensional relation and rapidly approaches the five dimensional relation. For $f + \bar{f} > 1$, the values for g lie on top of the five dimensional values within the resolution of the figure for $R = 1, 2, 10$. For $f + \bar{f} < 1$, the curves for $R = 0, 0.1, 0.2$ essentially coalesce while those for $R = 1, 2, 10$ have differing behavior from each other and from the four dimensional result. In general, for fixed $f + \bar{f}$, the radiative corrections vanish for smaller g values than that of the four dimensional case and even take on exponentially small values, $g \approx 2\pi R e^{-\pi R(f + \bar{f})}$, in the strong gauge coupling limit when $f + \bar{f} \ll 1$.

Being a trivial theory in four and above dimensions, the zero radiative correction line is an attractive relation between g and $f + \bar{f}$ in the sense that the renormalization group flow in the (g, f, \bar{f}) space tends towards the no radiative correction line, reaching it in the infrared at the single trivial point $g = 1$ and $f = \frac{1}{2} = \bar{f}$. This can be seen by considering the evolution of the ratio

$$\rho \equiv 1 + \xi \equiv \frac{\sinh[\pi R] \sinh[\pi R g]}{\sinh^2[\pi R \sqrt{f + \bar{f}}]}, \quad (2.6)$$

where $\xi|_{\varphi=0=\bar{\varphi}}=0$. Applying the Wilson renormalization group equations (2.2) and (1.3) gives

$$\begin{aligned} \frac{\partial \rho}{\partial t} = & (2\gamma g \pi R \coth[\pi R g])\rho - (1 - \gamma)\varphi\rho_{\varphi} - (1 - \gamma)\bar{\varphi}\rho_{\bar{\varphi}} \\ & + \frac{\pi R}{8\pi^2} \coth[\pi R g] \left[\rho_{\varphi\bar{\varphi}} - \frac{\rho_{\varphi}\rho_{\bar{\varphi}}}{\rho} \right]. \end{aligned} \quad (2.7)$$

Note that the anomalous dimension of the chiral fields, γ , can be expressed in terms of ξ as

$$\begin{aligned} 2\gamma = & -\frac{1}{8\pi^2} [\rho_{\varphi\bar{\varphi}} - \rho_{\varphi}\rho_{\bar{\varphi}}] \Big|_{\varphi=0=\bar{\varphi}} \\ = & -\frac{1}{8\pi^2} [\xi_{\varphi\bar{\varphi}} - \xi_{\varphi}\xi_{\bar{\varphi}}] \Big|_{\varphi=0=\bar{\varphi}}. \end{aligned} \quad (2.8)$$

For small ξ , g is slightly off the fixed relation no radiative correction curve. Hence, writing $g = g_0 + \Delta g$, where g_0 satisfies the no radiative correction condition, and expanding Eq. (2.5) yields

$$\Delta g = \frac{\tanh[\pi R g_0]}{\pi R} \xi. \quad (2.9)$$

Using Eq. (2.5), the evolution of ξ is given by

$$\begin{aligned} \frac{\partial \xi}{\partial t} = & -\varphi\xi_{\varphi} - \bar{\varphi}\xi_{\bar{\varphi}} + \frac{\pi R}{8\pi^2} \coth[\pi R g_0] \\ & \times [\xi_{\varphi\bar{\varphi}} - g_0\xi_{\varphi}\xi_{\bar{\varphi}}]_{\varphi=0=\bar{\varphi}} + O(\xi^2). \end{aligned} \quad (2.10)$$

As the theory flows into the infrared, $t > 0$, the values of the scaled fields, $\varphi \rightarrow e^{-t}\varphi$ and $\bar{\varphi} \rightarrow e^{-t}\bar{\varphi}$ become small and all quantities can be expanded in low powers of the fields. Thus for $t \gg 1$, $\xi \rightarrow 0$ exponentially according to the naive dimension of the power of the fields,

$$\xi \sim z_1 \varphi e^{-t} + \bar{z}_1 \bar{\varphi} e^{-t} + z_{11} \varphi \bar{\varphi} e^{-2t} + \dots \quad ; \quad t \rightarrow \infty. \quad (2.11)$$

Hence all renormalization group trajectories flow towards the no radiative correction relation as the theory evolves into the infrared, and they meet at the single trivial fixed point of the theory, $g = 1$ and $f = \frac{1}{2} = \bar{f}$ as $t \rightarrow \infty$. (Generally we are considering $f \neq 0 \neq \bar{f}$. For $f = 0 = \bar{f}$, the trivial infrared fixed point is simply $g = 1$. On the other hand if $g = 0$, then the action consists only of a free vector superfield, $f = 1/2 = \bar{f}$.)

III. THE PERTURBATIVE EFFECTIVE KÄHLER POTENTIAL

Similar conclusions can also be drawn from a one loop perturbative analysis of the model given by the action obtained by combining Eq. (1.2) with $\kappa = 0$ and Eq. (1.8). The one loop correction to the dimensionless effective Kähler potential is found to be

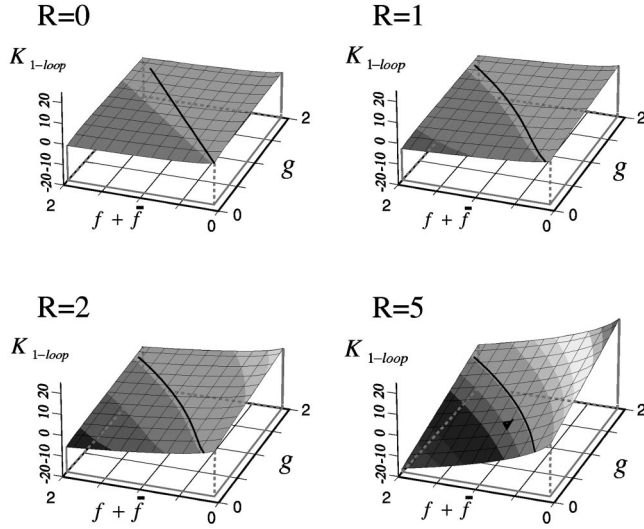


FIG. 2. The one loop radiative corrections to the Kähler potential for g and $f + \bar{f}$ values from 0.1 to 2.0 and for compactification radii $R=0,1,2,5$. Here $K_{1-loop} = -16\pi^2 \delta K_{1-loop}$. The darkened solid line is the no radiative correction relation.

$$\delta K_{1-loop} = -\frac{1}{16\pi^2} \int_0^1 d\xi \ln \left[\frac{\sinh \pi R \sqrt{g^2 \xi + r}}{\sinh \pi R \sqrt{\xi + r_0}} \frac{\sinh^2 \pi R \sqrt{\xi}}{\sinh^2 \pi R \sqrt{(f + \bar{f}) \xi}} \right], \quad (3.1)$$

where g, f, \bar{f} and r are specified dimensionless tree action terms while the integral is over the four dimensional momentum below the cutoff Λ , now scaled to 1. The fifth dimension K-K modes' propagation has been completely summed over in the loop.

In the four dimensional limit, $R \rightarrow 0$, with $r=0$, the momentum integral can be explicitly performed to yield the expected result [24–26]

$$\delta K_{1-loop}^{4D} = -\frac{1}{16\pi^2} \ln \left[\frac{g}{f + \bar{f}} \right], \quad (3.2)$$

which vanishes for the $N=2$ SUSY relationship between the metric and the effective gauge couplings: $g = f + \bar{f}$. For the case of a noncompact fifth dimension, $R \rightarrow \infty$, the sum over K-K modes again becomes an integral over the spatial momentum conjugate to the fifth dimension yielding

$$\begin{aligned} \frac{\delta K_{1-loop}}{2\pi R} &= -\frac{1}{32\pi^2} \int_0^1 d\xi \int_{-\infty}^{+\infty} \frac{dk}{2\pi} \ln \left[\frac{(g^2 \xi + k^2)(\xi + k^2)}{[(f + \bar{f}) \xi + k^2]^2} \right] \\ &= -\frac{1}{48\pi^2} [g - 2\sqrt{f + \bar{f}} + 1]. \end{aligned} \quad (3.3)$$

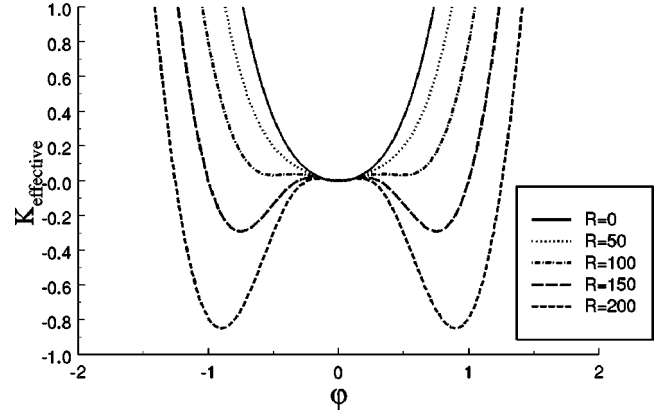


FIG. 3. $K_{\text{effective}}$ as a function of φ for compactification radii $R=0,50,100,150,200$. Note that the $R=0$ curve is simply the tree expression for K , Eq. (3.5).

This vanishes when $g - 2\sqrt{f + \bar{f}} + 1 = 0$, which is the identical result as was obtained in the Wilson renormalization group equation analysis. On the other hand, it should be noted that the relation between g and $f + \bar{f}$ for no radiative corrections in this perturbative case differs slightly from that obtained in the Wilson renormalization group analysis for moderate values of R . As seen in Fig. 2, the one loop contributions to the Kähler potential increase rapidly with increasing compactification radius R . Indeed, for large radii the quantum corrections are proportional to R . Already for R values of order 1 there are substantial deviations from the $R=0$ result. This is readily seen by the shape of the curve on each surface corresponding to the perturbative relation between g and $f + \bar{f}$ for the absence of radiative corrections. This $\delta K_{1-loop} = 0$ curve is displayed as the darkened solid line in the figure. The grid spacing on the surface corresponds to 0.2 changes in g and $f + \bar{f}$ while the shading of the surface corresponds to changes of 3 in the value of $K_{1-loop} = -16\pi^2 \delta K_{1-loop}$. For $R=0$, the K_{1-loop} surface is fairly flat, going from a minimum of -3 to a maximum value of $+3$, corresponding to fairly small radiative corrections to K . As R increases to 1, the K_{1-loop} values range from -4.79 to 5.67 while for $R=2$, the K_{1-loop} spread in values is from -7.9 to 10.2 . Thus the range in K_{1-loop} values increases as R increases. For $R=5$, the now quite substantial radiative corrections result in the K_{1-loop} surface ranging from a minimum of -18.3 to a maximum of $+24.8$.

The effects of large compact dimensions on the Kähler potential can be further demonstrated by considering an explicit tree level potential. In general, the radiative corrections are small due to the $1/16\pi^2$ loop factor. As the size of the extra dimension becomes the inverse of this order, however, the loop corrections become comparable with the tree level expression for the Kähler potential. For example, consider the $N=2$ SUSY holomorphic prepotential \mathcal{F} in four dimensions given by

$$\mathcal{F}(\varphi) = \frac{i}{2} (1 - 2a) \varphi^2 + \frac{ia}{b} (1 + b\varphi^2) \ln(1 + b\varphi^2), \quad (3.4)$$

where a and b are arbitrary parameters. The tree level Kähler potential derived from this is simply

$$K(\varphi, \bar{\varphi}) = \text{Im}(\mathcal{F}_\varphi \bar{\varphi}) = \varphi \bar{\varphi} [1 + a \ln(1 + b \varphi^2)(1 + b \bar{\varphi}^2)] \quad (3.5)$$

while the associated metric is

$$g(\varphi, \bar{\varphi}) = 1 + a \ln[(1 + b \varphi^2)(1 + b \bar{\varphi}^2)] + 2ab \left[\frac{\varphi^2}{1 + b \varphi^2} + \frac{\bar{\varphi}^2}{1 + b \bar{\varphi}^2} \right]. \quad (3.6)$$

Choosing the effective gauge couplings f and \bar{f} to be related to the metric g according to the $N=2$ SUSY relation $g=f+\bar{f}$, there are no radiative corrections to the Kähler potential in four dimensions. Taking $a=1=b$, the one loop effective Kähler potential

$$K_{\text{effective}} = K + \delta K_{1\text{-loop}} \quad (3.7)$$

is plotted in Fig. 3 for various values of the scalar fields, now taken as real and equal, $\varphi = \bar{\varphi}$. As expected, when R reaches the order of $16\pi^2$, the loop corrections to the Kähler potential are qualitatively different than the tree level values even for modest values of the fields. Note that as the scalar field strength continues to increase (beyond that shown in the figure), the effective Kähler potential is dominated by its tree value: $K_{\text{effective}} \sim \varphi \bar{\varphi} \ln(\varphi \bar{\varphi})$. This follows since the 1-loop radiative correction to K goes as g and hence as $\ln(\varphi \bar{\varphi})$.

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